New Reference-Neighbourhood Scalarization Problem for Multiobjective Integer Programming

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Abstract: Scalarization is a frequently used approach for finding efficient solutions that satisfy the preferences of the Decision Maker (DM) in multicriteria optimization. The applicability of a scalarization problem to solve integer multicriteria problems depends on the possibilities it provides for the decrease of the computing complexity in finding optimal solutions of this class of problems. This paper presents a reference-neighbourhood scalarizing problem, possessing properties that make it particularly suitable for solving integer problems. One of the aims set in this development has also been the faster obtaining of desired criteria values, defined by the DM, requiring no additional information by him/her. An illustrative example demonstrates the features of this scalarizing problem.

Keywords: Multicriteria optimization, integer programming, scalarization function.

1. Introduction

A lot of decision making problems in different areas of human activity are formulated and solved as integer optimization problems, and the dimension of such problems (objectives and/or variables) is constantly growing. Nowadays particular attention is paid to the explicit inclusion of several criteria in real applied models [2, 3, 4, 13].

The mathematic model, in which several conflicting and often incommensurable criteria (objective functions) are simultaneously optimized in the

set of feasible alternatives (solutions), is called a Multiobjective Optimization (MO) problem.

This article discusses the MultiObjective Integer Linear Programming (MOILP) problems, which can be represented in the following way:

(1) $$\text{"max"}\{f_k(x) = c^k x\}, \quad k \in K,$$

subject to:

(2) $$Ax \leq b,$$

(3) $$0 \leq x \leq d,$$

(4) $$x - \text{integer},$$

where x is an n-dimensional vector of variables, d is an n-dimensional vector of variable's upper bounds, A is an $m \times n$ matrix, b is the RHS vector and the vector c^i, $i = 1, ..., k$ represents the coefficients of the objective function $f_i(x)$.

Constraints (2)-(4) define the feasible set of the variables (solutions) X_1 . Problem (1)-(3) is a MultiObjective Linear Programming (MOLP) problem with continuous decision variables. Let us denote its feasible set of solutions by X_2 .

Let Z denotes the feasible region in the criteria space, i.e., the set of points $z \in R^k$, such that $z_k = f_k(x), k \in K, x \in X_1$.

Definition 1. $x^* \in X_1$ is an efficient or Pareto optimal solution iff there is no $x \in X_1$, such that $c^i x \geq c^i x^*$ for all i and $c^i x > c^i x^*$ for at least one i.

Definition 2. $x^* \in X_1$ is said to be weakly efficient/Pareto optimal solution iff there is no $x \in X_1$, such that $c^i x > c^i x^*$ for all i .

Although the term efficient is more often used for solutions x and the term Pareto Optimal (PO) – for points z, they can be used interchangeably.

Usually the set Z for problem (1)-(4) contains more than one PO points. The upper bound of the PO solutions in the criteria space can be estimated from the ideal criteria vector, which is defined through maximization of each one of the criteria (1) in the feasible set of solutions $X_1 - z_k^* = \max_{x \in X_1} f_k(x), \quad k \in K.$ In order to avoid the solution of k single-criterion integer problems, the ideal criteria vector of MOLP problem can be used, rounded to the greater integer value. The approximate evaluation of the lower bound of the efficient solutions in the criteria space $z_k^{\text{nad}}, \quad k \in K,$ can be obtained from the pay-off table.

Each PO point can be a solution of the MO problem from a mathematic point of view. In order to select the final solution of the problem, additional information is needed, provided by the so called Decision Maker (DM). This information reflects DM's preferences regarding the qualities of the solution sought [9]. Scalarization is a frequently used approach, in which the DM's preferences are used to find one (weak) PO solution. Applying different scalarizing problems for the same input information, reflecting DM's preferences, different efficient solutions can be found [1, 9, 10].

The formulation of the new reference-neighbourhood scalarizing problem is proposed with the intention to direct the process of search of (weak) PO solutions towards satisfying to the greatest extent of the aspiration values of a part of the criteria set by the DM. In addition, those that are pointed for improvement have greater importance compared to those, for which deterioration is admissible, or their directions of change are only given.

2. Description of the new reference-neighbourhood scalarizing problem

When solving a MOILP problem using an interactive algorithm, based on a classification-oriented scalarizing problem (such as the reference-neighbourhood scalarizing problem), the DM evaluates and compares the values of each one of the criteria z_k, $k \in K$, in the currently found (weak) efficient solutions. In case the DM wants to search for another solution, he/she presents new preferences for desired or feasible alterations in the values of a part or of all the criteria.

The DM might know better some of the criteria and could express this intention with the help of explicit values, by which they must be improved − Δ_k, $k \in K^\geq$, or feasible values, by which they will be deteriorated − σ_k, $k \in K^\leq$. In other words, for this criteria group the DM is able to define the aspiration levels $\bar{f}_k$, $k \in K^\geq \cup K^\leq$, that have to be achieved in the solution found. For another criteria group the DM might accept the values obtained within certain intervals and set limiting values about them in the form of intervals $z_k - t_k^- \leq f_k(x) \leq z_k + t_k^+$, $k \in K^{><}$, or preservation of the already obtained value z_k, $k \in K^=$. The DM might not have any notion of the possible values of some criteria (especially at the initial iterations of the process of the solution selection) and he/she may only decide that some of the criteria must be improved on the account of deterioration of others, with respect to the current solution considered z_k, $k \in K^>$ or $k \in K^<$.

In the new reference-neighbourhood scalarization problem proposed, the process of new solution search is accomplished on the basis of the information about the desired improvement of at least one criterion ($K^\geq \cup K^> \neq \varnothing$) and about the acceptable deterioration of some criteria ($K^\leq \cup K^< \neq \varnothing$). In order to speed up the choice of the most preferred solution, the DM's preferences must be reflected as precisely as possible in the scalarizing problem applied. Obviously, the changes of the criteria belonging to the classes $K^\geq \cup K^\leq$ are the most desired by the DM. For the criteria belonging to classes $K^> \cup K^<$ the changes are recommendable, and for those to classes $K^= \cup K^{><}$, they are only restrictive. This might be reflected by means of determination of different importance-weights of the criteria belonging to different classes − comparing wished and recommendable criteria

changes – greater weight must be given to obtain the wished changes, while the reservation ones must be included in the problem constraints.

We suggest the following scalarizing problem, New Reference-Neighbourhood Problem (NRNP1) for finding a weak PO solution of MOILP problem:

(5)
$$\min_{x \in X_1} S(x) = \min_{x \in X_1} \left(\max_{k \in K^{\geq} \cup K^{\leq}} \left(\frac{\omega_k \left(\overline{f}_k - f_k(x) \right)}{\left| z_k^{**} - z_k \right|} \right) + \right.$$
$$\left. + \max \left(\max_{k \in K^{>}} \left(\frac{\omega_k \left(z_k^{**} - f_k(x) \right)}{\left| z_k^{**} - z_k \right|} \right), \max_{k \in K^{<}} \left(\frac{\omega_k \left(z_k - f_k(x) \right)}{\left| z_k^{**} - z_k \right|} \right) \right) \right)$$

subject to:

(6)
$$f_k(x) \geq z_k, \quad k \in K^{>} \cup K^{\geq} \cup K^{=} \ ,$$

(7)
$$z_k - t_k^{-} \leq f_k(x) \leq z_k + t_k^{+} , \quad k \in K^{><},$$

(8)
$$x \in X_1,$$

where z_k is the value of the criterion with an index $k \in K$ for the current preferred solution, $\overline{f}_k$ is the desired (aspiration) level of the criterion with an index $k \in K^{\geq} \cup K^{\leq}$,

$$\overline{f}_k = \begin{cases} z_k + \Delta_k, \text{ if } k \in K^{\geq} , \\ z_k - \delta_k, \text{ if } k \in K^{\leq} . \end{cases}$$

The first term of the scalarizing problem is introduced with the purpose to minimize the maximal deviation of the corresponding criteria from the aspiration levels being set, and the second one – to minimize the maximal deviation from the ideal value of the criteria, which are intended to be improved and to minimize the maximal deviation from the current solution found for the criteria with feasible deterioration.

The influence of the weighting coefficients in the achievable functions of the scalarizing problems has been investigated in many publications [6, 7, 12]. The use of the coefficient ω_k in the objective function of the scalarizing problem is with the designation to increase the influence of the criteria, for which the DM has set values that must be achieved. In order to evaluate the weight ω_k, it is proposed to use the total number of the criteria and the number of the criteria, distributed in each one of the classes, in conformance with the defined DM's preferences of the new compromise solution being searched for

$$(9) \qquad \omega_k = \begin{cases} |K|, k \in K^{\geq}, \\ |K| - |K^{\geq}|, k \in K^{\leq}, \\ |K| - |K^{\geq}| - |K^{\leq}|, k \in K^{>}, \\ |K| - |K^{\geq}| - |K^{\leq}| - |K^{>}|, k \in K^{<}, \end{cases}$$

$|..|$ denotes the cardinality of the corresponding set.

Hereby the problem does not require additional ranking of the criteria by the DM with respect to his/her wish for achievability of the preferences set [6]. This information is indirectly derived from the classification of the criteria into groups. The criteria in the group of wished alterations ($K^{\geq} \cup K^{\leq}$) are of the highest priority, followed by the recommendable alterations ($K^{>} \cup K^{<}$).

The proposed scalarizing problem NRNP1 possesses the feature that the current solution of MOILP problem is a feasible solution (constraints (6)-(8) being satisfied) of the scalarizing problem, formulated at the next iteration. This is a very important feature, because finding a feasible solution of the integer problems is an NP-hard problem as well.

In addition, with the help of his/her local preferences, the DM can set not only a reference point (defined by the desired values of the criteria), but a reference area as well (defined by intervals and directions of alteration). It could either be very narrow or considerably extended in case the DM has prescribed free improvement or free deterioration for most of the criteria.

Theorem 1. The solution of NRNP1 scalarizing problem is a Pareto optimal solution of MOILP problem.

P r o o f:

Let $K^{\geq} \neq \varnothing$ and/or $K^{>} \neq \varnothing$ and let $x^0 \in X_1$ be the optimal solution of the scalarizing problem NRNP1. Then $S(x^0) \leq S(x)$, for each $x \in X_1$, satisfying the constraints (6)-(7).

Let us assume that $x^0 \in X_1$ is not a weak Pareto optimal solution of the initial MOILP problem. In this case there must exist another $x' \in X_1$, for which:

$$(10) \qquad f_k(x') > f_k(x^0) \quad \text{for } k \in K$$

and x' is a feasible solution of NRNP1 problem, i.e., it satisfies the constrains:

$$f_k(x') \geq z_k, \quad k \in K^{>} \cup K^{\geq} \cup K^{=},$$
$$z_k - t_k^- \leq f_k(x') \leq z_k + t_k^+, \quad k \in K^{><},$$

After transformation of the objective function $S(x')$ of the scalarizing problem NRNP1, using inequalities (10), the following relation is obtained:

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$$S(x') = \max_{k \in K^{\geq} \cup K^{\leq} \cup K^{=}} \left(\frac{\omega_k \left(\overline{f}_k - f_k(x') \right)}{\left| z_k^{**} - z_k \right|} \right) +$$

$$+ \max \left(\max_{k \in K^{<}} \left(\frac{\omega_k \left(f_k - f_k(x') \right)}{\left| z_k^{**} - z_k \right|} \right), \max_{k \in K^{>}} \left(\frac{\omega_k \left(f_k^{*} - f_k(x') \right)}{\left| z_k^{**} - z_k \right|} \right) \right) =$$

$$= \max_{k \in K^{\geq} \cup K^{\leq}} \left(\omega_k \left(\frac{\left(\overline{f}_k - f_k(x^0) \right)}{\left| z_k^{**} - z_k \right|} + \frac{\left(f_k(x^0) - f_k(x') \right)}{\left| z_k^{**} - z_k \right|} \right) \right) +$$

$$+ \max \left(\max_{k \in K^{>}} \left(\omega_k \left(\frac{z_k^{**} - f_k(x^0)}{\left| z_k^{**} - z_k \right|} + \frac{f_k(x^0) - f_k(x')}{\left| z_k^{**} - z_k \right|} \right) \right), \right.$$

$$\left. \max_{k \in K^{<}} \left(\omega_k \left(\frac{z_k - f_k(x^0)}{\left| z_k^{**} - z_k \right|} + \frac{f_k(x^0) - f_k(x')}{\left| z_k^{**} - z_k \right|} \right) \right) \right) <$$

$$< \max_{k \in K^{\geq} \cup K^{\leq}} \left(\frac{\omega_k \left(\overline{f}_k - f_k(x^0) \right)}{\left| z_k^{**} - z_k \right|} \right) +$$

$$+ \max \left(\max_{k \in K^{>}} \left(\frac{\omega_k \left(z_k^{**} - f_k(x^0) \right)}{\left| z_k^{**} - z_k \right|} \right), \max_{k \in K^{>}} \left(\frac{\omega_k \left(z_k - f_k(x^0) \right)}{\left| z_k^{**} - z_k \right|} \right) \right) = S(x^0).$$

From the above given it follows that $S(x') < S(x^0)$. This is in contradiction with the assumption that $x^0 \in X_1$ is an optimal solution of NRNP1 scalarizing problem; therefore $x^0 \in X_1$ is a weak Pareto optimal solution. ∎

The scalarizing problem NRNP1 may be presented in the form of an equivalent mixed integer problem at the expense of additional constraints and real variables [15]. This Equivalent of NRNP1 problem (NRNP1E) is with a differentiable objective function and can be solved with the help of the traditional methods of single-criterion optimization. NRNP1E scalarizing problem has the following form:

(11) $$\min_{x \in X_1} \left(\alpha + \beta \right)$$

subject to:

(12) $$\alpha \geq \frac{\omega_k \left(\overline{f}_k - f_k(x) \right)}{\left| z_k^{**} - z_k \right|}, \quad k \in K^{\geq} \cup K^{\leq},$$

(13) $$\beta \geq \frac{\omega_k \left(z_k^{**} - f_k(x) \right)}{\left| z_k^{**} - z_k \right|}, \quad k \in K^{>},$$

$$(14) \qquad \beta \geq \frac{\omega_k\left(z_k - f_k(x)\right)}{\left|z_k^{**} - z_k\right|}, \quad k \in K^<,$$

$$(15) \qquad f_k(x) \geq z_k, \quad k \in K^> \cup K^\geq \cup K^= ,$$

$$(16) \qquad z_k - t_k^- \leq f_k(x) \leq z_k + t_k^+, \quad k \in K^{><},$$

$$(17) \qquad x \in X_1,$$

$$(18) \qquad \alpha, \beta - \text{arbitrary}.$$

As it can be seen, problems NRNP1 and NRNP1E have the same feasible set of solutions $x \in X_1$. The following assertion can be proved:

Theorem 2. The optimal values of the objective functions of scalarizing problems NRNP1 and NRNP1E are equal.

P r o o f:

Inequality (12) $\alpha \geq \dfrac{\omega_k\left(\bar{f}_k - f_k(x)\right)}{\left|z_k^{**} - z_k\right|}$, must be satisfied for

every $k \in K^\geq \cup K^\leq$, then it is also true that

$$(19) \qquad \alpha \geq \max_{k \in K^\geq \cup K^\leq}\left(\frac{\omega_k\left(\bar{f}_k - f_k(x)\right)}{\left|z_k^{**} - z_k\right|}\right).$$

Similarly it follows from (13) that

$$(20) \qquad \beta \geq \max_{k \in K^>}\left(\frac{\omega_k\left(z_k^{**} - f_k(x)\right)}{\left|z_k^{**} - z_k\right|}\right),$$

and from (14):

$$(21) \qquad \beta \geq \max_{k \in K^<}\left(\frac{\omega_k\left(z_k - f_k(x)\right)}{\left|z_k^{**} - z_k\right|}\right),$$

For β from (20) and (21), the following can be written:

$$(22) \qquad \beta \geq \max\left(\max_{k \in K^>}\left(\frac{\omega_k\left(z_k^{**} - f_k(x)\right)}{\left|z_k^{**} - z_k\right|}\right), \max_{k \in K^<}\left(\frac{\omega_k\left(z_k - f_k(x)\right)}{\left|z_k^{**} - z_k\right|}\right)\right).$$

If the left and right sides of inequalities (19) and (22) are summed, it will be obtained:

$$\alpha + \beta \geq \max_{k \in K^\geq \cup K^\leq}\left(\frac{\omega_k\left(\bar{f}_k - f_k(x)\right)}{\left|z_k^{**} - z_k\right|}\right) +$$

$$(23)$$

$$+ \max\left(\max_{k \in K^>}\left(\frac{\omega_k\left(z_k^{**} - f_k(x)\right)}{\left|z_k^{**} - z_k\right|}\right), \max_{k \in K^<}\left(\frac{\omega_k\left(z_k - f_k(x)\right)}{\left|z_k^{**} - z_k\right|}\right)\right).$$

Let x' be the optimal solution of NRNP1E problem. Then

$$\min_{x \in X_1} (\alpha + \beta) = \left(\max_{k \in K^{\geq} \cup K^{\leq}} \left(\frac{\omega_k \left(\overline{f}_k - f_k(x') \right)}{\left| z_k^{**} - z_k \right|} \right) + \right.$$

(24)

$$\left. + \max \left(\max_{k \in K^{>}} \left(\frac{\omega_k \left(z_k^{**} - f_k(x') \right)}{\left| z_k^{**} - z_k \right|} \right), \max_{k \in K^{<}} \left(\frac{\omega_k \left(z_k - f_k(x') \right)}{\left| z_k^{**} - z_k \right|} \right) \right) \right),$$

because in the opposite case $\alpha + \beta$ might be still decreasing.

Then the right side of (24) can be written also as

$$\min_{x \in X_1} (\alpha + \beta) = \min_{x \in X_1} \left(\max_{k \in K^{\geq} \cup K^{\leq}} \left(\frac{\omega_k \left(\overline{f}_k - f_k(x) \right)}{\left| z_k^{**} - z_k \right|} \right) + \right.$$

$$\left. + \max \left(\max_{k \in K^{>}} \left(\frac{\omega_k \left(z_k^{**} - f_k(x) \right)}{\left| z_k^{**} - z_k \right|} \right), \max_{k \in K^{<}} \left(\frac{\omega_k \left(z_k - f_k(x) \right)}{\left| z_k^{**} - z_k \right|} \right) \right) \right),$$

which proves the theorem. ∎

The scalarizing problem NRNP1E is a mixed integer problem, hence it is an NP-hard problem. It also possesses the property that a PO solution, found at a previous iteration, is a feasible solution of the current NRNP1E, which considerably reduces the computing efforts to find an optimal solution.

3. An illustrative example

A simple bi-criteria linear integer example is considered, which demonstrates the features of the offered new reference-neighbourhood scalarizing problem NRNP1:

$$\max \quad f_1(x) = -4x_1 + x_2,$$
$$\max \quad f_2(x) = x_1 - 2x_2$$

subject to:

$$2x_1 + x_2 \leq 48,$$
$$x_1 + 3x_2 \leq 72,$$
$$x_1 \geq 4,$$
$$x_2 \geq 4,$$
$$x_1, x_2 - \text{integer}.$$

Before starting the process of choice of the most preferred solution, it is useful to present to the DM the best and the worst value of each one of the criteria in the feasible set of the variables. Since these values are intended to orient the DM in the possible limits of criteria alteration, approximate values are used. In order to obtain an approximate evaluation of the ideal point and to avoid the solution of a more difficult integer problem, each one of the criteria is optimized on the feasible continuous set of solutions. The following results are obtained:

$f_1^{**}(4; 22.67) = 6.67$, assume $z_1^{**} = 7$ and $f_2^{**}(22; 4) = 14$, $z_2^{**} = 14$. The evaluation of Nadir values, obtained from the pay-off table, is the following: $z_1^{\text{nad}} = -84$ and $z_2^{\text{nad}} = -42$. In order to find an initial PO solution, the so called neutral compromise solution [15] may be used. The aspiration levels of the criteria are set, defined by: $\bar{f}_k = \dfrac{z_k^{**} + z_k^{\text{nad}}}{2}$, $\bar{f}_1 = -38, \bar{f}_2 = -14$. As a current start solution, that will be subjected to improvement, an arbitrary selected, worse than $\bar{f}_k$ solution may be chosen, for example: $z_1 = -40$ and $z_2 = -20$. Then $K^{\geq} = \{1; 2\}$ and $\omega_1 = \omega_2 = 2$. After solving NRNP1E scalarizing problem for these parameters, we find an initial PO solution $z_1^0 = f_1^0(6; 4) = -20$, $z_2^0 = f_2^0(6; 4) = -2$.

In case the DM wants to improve the first criterion on the account of deterioration of the second one, a new scalarizing problem NRNP1E is formed with the following parameters: $K^{>} = \{1\}$, $K^{<} = \{2\}$ and $\omega_1 = 2$, $\omega_2 = 1$. Through the weighting coefficient $\omega_1 = 2$, higher priority is given to the desired improvement of the first criterion with respect to a feasible deterioration of the second one. After solution of NRNP1E problem with DM's preferences thus defined, a new PO solution is found: $z_1^1 = f_1^1(4; 10) = -6$, $z_2^1 = f_2^1(4; 10) = -16$. It is noticed that in this solution the first criterion is considerably improved. If for finding a new current solution RNP1E scalarizing problem [5] is used, we obtain another PO solution $z_1^2 = f_1^2(4; 7) = -9$, $z_2^2 = f_2^2(4; 7) = -10$, in which the desired improvement of the first criterion is smaller.

Starting from an initial PO solution ($z_1^0 = -20$, $z_2^0 = -2$), taking in mind the approximate interval of the criteria alteration, the DM might prefer to seek a solution that improves the second criterion on the account of a feasible deterioration of the first one. He/she sets aspiration levels: $\bar{f}_1 = -40, \bar{f}_2 = 7$. Then $K^{\leq} = \{1\}$, $K^{\geq} = \{2\}$ and $\omega_1 = 1$, $\omega_2 = 2$. For these parameters, the PO solution found with NRNP1 problem is: $z_1^3 = f_1^3(13; 4) = -48$, $z_2^3 = f_2^3(13; 4) = 5$. The solution obtained is closer to the aspiration value of the criterion, for which improvement is sought, in comparison to attaining an aspiration value of the criterion, for which a feasible level of deterioration is set. The PO solution obtained with the help of RNP1E scalarizing problem [5] for the same defined preferences is: $z_1^4 = f_1^4(11; 4) = -40$, $z_2^4 = f_2^4(11; 4) = 3$.

Fig. 1 presents the feasible area in the criteria space, as well as the PO solutions obtained. This test example demonstrates one of the features of the new reference-neighbourhood scalarizing problem NRNP1, namely that the priority of the desired criteria improvements in relation to other set alterations, are automatically given, and the PO solutions found satisfy the DM's intentions for criteria improvement. In this way the convergence of the process, selecting a compromise solution, satisfactory for the DM, is accelerated.
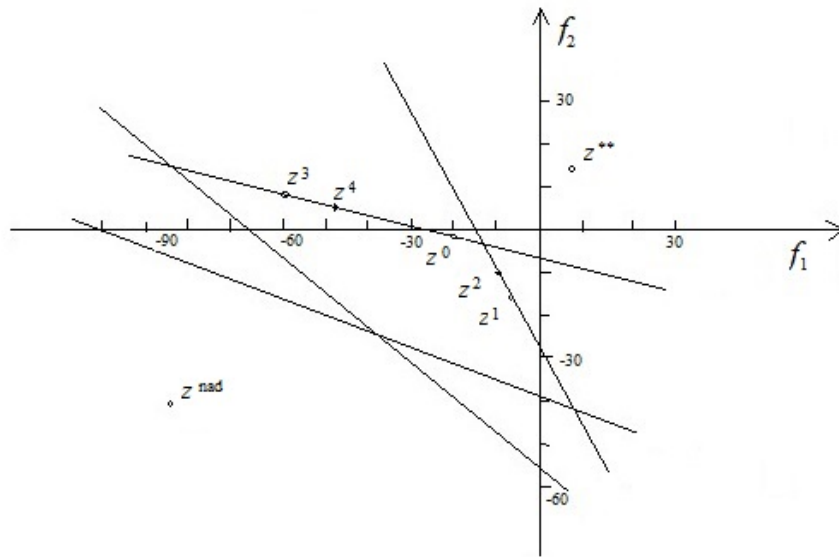
Fig. 1

4. Conclusion

The presented reference-neighborhood scalarizing problem NRNP1 is designed to set priorities in satisfying DM's preferences. They direct the process of search of PO solutions in a way that will satisfy as close as possible the aspiration levels set by the DM.

The scalarizing problem NRNP1 (as well as its equivalent problem NRNP1E) preserves the feature of the scalarizing problems of this class. Namely, the current preferred solution at the previous iteration is a feasible solution for the scalarizing problem, solved at the next iteration. The possibilities for setting DM's preferences about the allowable intervals of change or keeping the reached values for some criteria, can considerably tighten the feasible domain of the integer scalarizing problem, which has to be solved, and thus reduce the time for finding a new solution.

Considering the development of interactive algorithms for solving multiobjective integer optimization problems, based on this problem formulation, strategies for search of continuous or approximate solutions during the initial iterations could be applied. At these iterations the DM directs the search process to this part of the Pareto front, where the criteria values are close to the values satisfactory for the DM. After that, the constraint for integer values is applied, so that integer PO solutions are found.

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